

# ANALYSIS OF THE CAPILLARY-TRANSPORT CHARACTERISTICS OF METAL-FIBER STRUCTURES

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On the basis of experimental data and model representations, the article analyzes the capillary-transport characteristics of metal-fiber structures (MFS).

The designing of heat pipes, various systems of porous cooling, capillary-arterial systems, etc., requires a detailed analysis of the transport properties of porous materials. The authors of [1-3] present the results of the experimental investigation of the fundamental characteristics determining the capillary-transport properties of porous metal-fiber structures (MFS) and data on the kinetics of capillary absorption.

On the basis of experimental data and model representations we develop, make more accurate, and generalize the tenets and results expressed in [1-3] for the purpose of further developing the scientific foundations of rationally designing capillary-transport systems.

1. Coefficient of Liquid Permeability. The mathematical expressions characterizing the known analytical models of permeability have identical construction of the form  $K_{KC}/d_B^2 = \psi(\Pi)$ , i.e., the dimensionless coefficient of permeability  $K_{KC}/d_B^2$  is some single-valued function or other of the porosity.

Figure 1 shows the best-known models of permeability for materials made up of cylindrical particles: the model of the hydraulic radius, retarding and cell models (for flows parallel and perpendicular to the cylinder axes). A comparison of the results of calculations based on them indicates that there are large divergences in the numerical values of permeability. Also plotted in the figure are points corresponding to the experimental data obtained by the authors of [1] in the region of "regular" flow perpendicularly to the plane of felting of the MFS. The experimental data are stratified according to the magnitude of the ratio  $l_B/d_B$ .

Obviously, the initial structural parameters ( $\Pi$ ,  $d_B$ ,  $l_B$ ) have an additional effect on the permeability of the MFS. This is caused by the peculiarities of the arrangement of the real structure which cannot be taken into account within the framework of the analytical models. These peculiarities consist in some change in the structure of the pore space and in the orientation of the fibers when the porosity changes. The MFS are fairly complex systems with remote order. The production technology determines the predominant orientation of the fibers in the felting planes perpendicular to the direction of the forming force (direction of pressing). However, with increasing porosity, especially in the region of its limit values, the proportion of fibers orientated at angles other than a right one to the forming force increases, and this causes an additional increase of the permeability. Moreover, when the porosity in the region near the boundary region increases, the inhomogeneity of the structure increases. Inhomogeneous structures are characterized by multiple sudden widening and narrowing of the pore channels causing an increase of hydraulic resistance.

The analytical models establish the increasing nature of the dependence of permeability on porosity. The mentioned peculiarities of real structures that have the opposite effect on the change of the function  $\partial K_{KC}/\partial \Pi = \gamma(\Pi)$  do not change its plus sign; this is indicated by the experimental data.

All models were constructed without regard to the filtration effect, and therefore they correspond more to the real pattern of gas permeability. The filtration effect has an additional influence on the coincidence of the experimental results with the model representations.

Figure 1 shows that the best agreement with the experimental data is found for the cell model for flow perpendicular to the cylinder (fiber) axes. By its initial prerequisites and analysis it is closest to the real filtering conditions of MFS. The mathematical expression of the model has the form [4]

$$K_{KC} = \frac{d_B^2}{16} \left[ \frac{1 - \Pi}{1 + (1 - \Pi)^2} - \frac{1 + \ln(1 - \Pi)}{2(1 - \Pi)} \right]. \quad (1)$$

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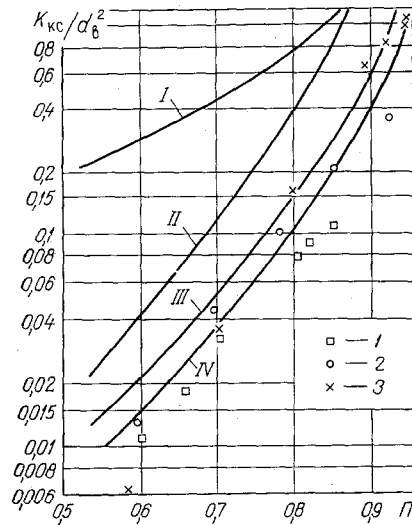


Fig. 1

Fig. 1. Comparison of the experimental values of dimensionless permeability of MFS with the analytical models: 1)  $l_B/d_B = 43$ ; 2) 75; 3) 150; I) retarding model; II) model of hydraulic radius; III) cell (flow parallel to the cylinder axes); IV) cell (flow perpendicular to the cylinder axes).

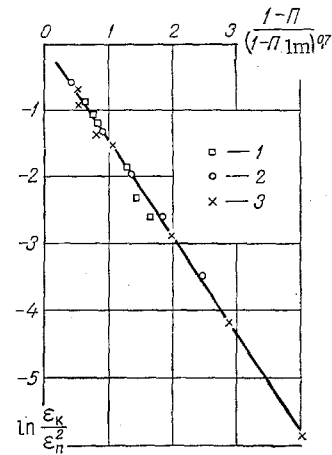


Fig. 2

Fig. 2. Generalization of the experimental data on liquid permeability of MFS: 1)  $l_B/d_B = 43$ ; 2) 75; 3) 150; line: generalization.

We adopt (1) as the basis and take into account the discovery of the peculiarities of real structures with the aid of some function of the argument establishing the degree of closeness of the arrangement of the given porosity of the structure to the structure of the maximally attainable porosity with the same size fibers. Such a magnitude is the ratio  $(1-\Pi)/(1-\Pi_{lm})$  which is numerically equal to the degree of compression in pressing the structure of the maximally attainable porosity so that it attains the structure of specified porosity. Then the expression for permeability is written as follows:

$$K_{KC} = \frac{d_B^2}{16} \left[ \frac{1-\Pi}{1+(1-\Pi)^2} - \frac{1+\ln(1-\Pi)}{2(1-\Pi)} \right] f[(1-\Pi), (1-\Pi_{lm})]. \quad (2)$$

Being dependent on the fiber dimensions, the maximum porosity may be regarded as an additional characteristic of the structure, in addition to  $l_B$ ,  $d_B$ , and  $\Pi$ . The magnitude of the maximum porosity of the MFS also affects the features of the production technology (especially the method of felting), the dimensions of the specimen, the state of the fibers, etc. The largest value of maximum porosity (maximally attainable porosity), causing the greatest inhomogeneity of the structure, can be obtained in aerial felting of straight fibers. The values of the maximally attainable porosity under conditions of aerial felting of straight fibers are approximated by the expression

$$\Pi_{lm} = \exp \left( -6 \frac{d_B}{l_B} \right), \quad 15 \leq \frac{l_B}{d_B} \leq 150, \quad (3)$$

which generalizes the experimental data including those from [15] with an error of up to  $\pm 4\%$ .

From Eq. (2) the numerical values of the function  $f[(1-\Pi), (1-\Pi_{lm})]$  were obtained by comparing the experimental values of permeability [1] with those calculated by the cell model (1). The obtained numerical values were examined in dependence on the ratio  $(1-\Pi)/(1-\Pi_{lm})$ . As a result of the analysis we established a linear functional correlation in the coordinates  $(1-\Pi)/(1-\Pi_{lm})^{0.7}$ ,  $\ln(\epsilon_K/\epsilon_{II}^2)$  (Fig. 2), where  $\epsilon_K$  is the ratio of the experimental value of the coefficient of permeability to the corresponding coefficient calculated by (1), and  $\epsilon_{II} = (1-\Pi)/(1-\Pi_{lm})$ . On the basis of the found generalization, the expression for liquid permeability of fully saturated porous MFS is written in the form

$$K_{KC} = \frac{d_B^2}{16} \left[ \frac{1-\Pi}{1+(1-\Pi)^2} - \frac{1+\ln(1-\Pi)}{2(1-\Pi)} \right] \left( \frac{1-\Pi}{1-\Pi_{lm}} \right)^2 \exp \left[ -1.45 \frac{1-\Pi}{(1-\Pi_{lm})^{0.7}} \right], \quad (4)$$

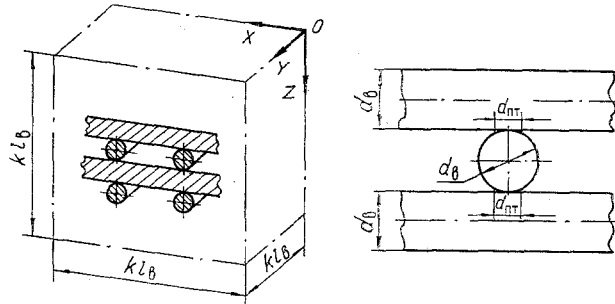


Fig. 3. Diagram concerning the problem of determining capillary pressure.

where  $0.55 \leq \Pi \leq 0.95$ , and  $\Pi_{lm}$  is determined by formula (3). The difference between the experimental data and the result of calculation by (4) does not exceed 20%.

According to (4) the coefficient  $K_{KC}$  is a function of three independent variables:  $\Pi$ ,  $d_B$ ,  $l_B$ . The partial derivatives of this function with respect to  $\Pi$  and  $d_B$  in the investigated range of structural parameters are positive magnitudes. Consequently, when  $d_B$  and  $l_B$  are fixed values, the coefficient  $K_{KC}$  increases with increasing  $\Pi$ ; with  $\Pi$  and  $l_B$  being fixed values, the coefficient  $K_{KC}$  increases with increasing  $d_B$ . Furthermore, as a result of investigation of the function it was established that the partial derivative  $\partial K_{KC} / \partial l_B$  is a magnitude with alternating sign. For  $(1 - \Pi) / (1 - \Pi_{lm})^{0.7} < 2$   $\partial K_{KC} / \partial l_B > 0$ , and for  $(1 - \Pi) / (1 - \Pi_{lm})^{0.7} > 2$   $\partial K_{KC} / \partial l_B < 0$ . For

$$(1 - \Pi) / (1 - \Pi_{lm})^{0.7} = 2 \quad (5)$$

the condition of existence of an extremum of the dependence of  $K_{KC}$  on  $l_B$  is fulfilled, i.e.,  $\partial K_{KC} / \partial l_B = 0$ . The solution of Eq. (5) with a view to (3) and with respect to  $l_B$  yields an expression for  $l_B$  at which the coefficient  $K_{KC}$  has its maximum value (with fixed values of  $\Pi$  and  $d_B$ ):

$$l_B^* = - \frac{6d_B}{\ln [1 - 0.37(1 - \Pi)^{1.43}]} \quad (6)$$

Formula (6) is correct for  $0.55 \leq \Pi \leq 0.95$  and  $15 \leq l_B / d_B \leq 150$ .

2. Capillary Pressure. An approach is known where in determining the capillary pressure  $p_K$  induced by complex capillary porous systems the Laplace equation is used in which the characteristic dimension is the so-called effective pore diameter  $D_e$ . This magnitude is determined experimentally on the basis of a model of an ideal porous medium. In the present work we suggest, as a result of an analysis, the dependence for capillary pressure in which we do not use the nominal magnitude of the effective pore diameter but take into account only the initial structural parameters.

We will examine a model of an MFS (Fig. 3), removing part of the volume of the structure in the form of a cube with the side  $kl_B$  ( $k < 1$ ). The fibers are sintered together. For capillary pressure reduced to the plane perpendicular to the plane XOY we can write (with  $\cos \theta = 1$ )

$$p_K = c \frac{\sigma U_*}{F_*} \quad (7)$$

where  $U_*$  is the smallest summary length (taking capillary hysteresis into account) of the phase interfaces in the plane parallel to the plane XOZ;  $F_*$ , clear cross section of the liquid in that same plane.

We present the expression for  $U_*$  in the form

$$U_* = 0.5n_B(\pi d_B - 2d_{nT}) \quad (8)$$

where  $n_B$  is the number of fibers in the removed volume:

$$n_B = \frac{4}{\pi} (1 - \Pi) \left( \frac{kl_B}{d_B} \right)^2 \quad (9)$$

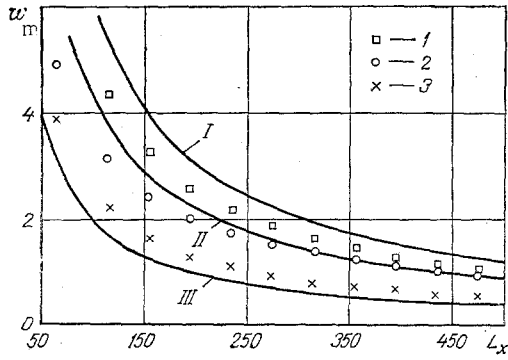


Fig. 4. Comparison of the experimental (1-3) and theoretical (I-III) values of the speed of the impregnation front for water and acetone with  $g \sin \varphi = 0$ ; 1, I)  $l_B/d_B = 43$ ; II = 0.792; 2, II) 75 and 0.794; 3, III) 150 and 0.79, respectively.  $w_m$ , mm/sec;  $L_x$ , mm.

Then we obtain for  $U_*$

$$U_* = 2(1 - \Pi) \frac{(kl_B)^2}{d_B} \left( 1 - \frac{2}{\pi} \frac{d_{\Pi T}}{d_B} \right). \quad (10)$$

As a result of metallographic analysis of the MFS it was established that the relative size of the contact spot  $d_{\Pi T}/d_B$  is proportional to the ratio  $l_B/d_B$  to the 0.5th power and inversely proportional to the porosity to the 0.1th power, i. e.,

$$\frac{d_{\Pi T}}{d_B} = c_1 \left( \frac{l_B}{d_B} \right)^{0.5} \Pi^{-0.1}. \quad (11)$$

Since in formula (3)  $6d_B/l_B \leq 0.4$ , in expanding  $\exp(-6d_B/l_B)$  into a power series we may confine ourselves to two of its terms and establish an approximate proportionality between the ratio  $l_B/d_B$  and the value of  $(1 - \Pi_{lm})^{-1}$ :

$$\frac{l_B}{d_B} \approx \frac{6}{1 - \Pi_{lm}}. \quad (12)$$

Taking (11) and (12) into account, formula (10) assumes the form

$$U_* = 2(1 - \Pi) \frac{(kl_B)^2}{d_B} [1 - 1.56c_1 \Pi^{-0.1} (1 - \Pi_{lm})^{-0.5}]. \quad (13)$$

On the basis of a numerical analysis we can show that the expression in square brackets in formula (13) in the investigated range of structural parameters of MFS is practically proportional to the complex  $[\Pi(1 - \Pi_{lm})^{0.5}]$ . Then

$$U_* = c_2 \frac{(kl_B)^2}{d_B} \Pi (1 - \Pi) (1 - \Pi_{lm})^{0.5}. \quad (14)$$

The clear cross section is

$$F_* = (kl_B)^2 \Pi. \quad (15)$$

If we substitute (14) and (15) into (7), we obtain

$$p_K = c' \frac{\sigma}{d_B} (1 - \Pi) (1 - \Pi_{lm})^{0.5}. \quad (16)$$

The coefficient  $c'$  was found as a result of substituting the experimental data into (16); it was  $c' = 35$ . The experimental values of  $p_K$  were determined from the experimental values of the heights of capillary equilibrium of water and acetone in specimens of MFS presented in [3].

Thus the expression for capillary pressure induced by an MFS has the form

$$p_K = 35 \frac{\sigma}{d_B} (1 - \Pi) (1 - \Pi_{lm})^{0.5}. \quad (17)$$

where  $0.6 \leq \Pi \leq 0.95$ , and the values of  $\Pi l_{\text{m}}$  are determined by formula (3). For  $\cos \Theta < 1$ , the right-hand side of expression (17) has to contain the cofactor of  $\cos \Theta$ . The difference between the experimental data of [3] and the calculation result by (17) does not exceed  $\pm 15\%$ .

An analysis of formula (17) as a function of the independent variables  $\Pi$ ,  $d_{\text{B}}$ ,  $l_{\text{B}}$  shows that the partial derivatives  $\partial p_{\text{K}}/\partial \Pi$ ,  $\partial p_{\text{K}}/\partial d_{\text{B}}$  and  $\partial p_{\text{K}}/\partial l_{\text{B}}$  are negative magnitudes, i.e., with two constant arguments, the function of the third argument is decreasing.

**3. Kinetics of Capillary Pressure.** In absorption of a liquid by a dry porous specimen with length  $L$  and the conditions  $g \sin \varphi = 0$ , we can write for the cross section at the distance  $L_{\text{x}}$  from the beginning of absorption in the first approximation the known equation of equilibrium of pressures

$$p_{\text{K}} = \Delta p_{\text{l}}. \quad (18)$$

Here we neglect the evaporation of liquid from the surface and the pressure gradient due to a change in impulse.

The process of capillary absorption is essentially a non-steady-state process. Since the literature contains information on the applicability of Darcy's equation to non-steady-state processes [6], we write

$$\Delta p_{\text{l}} = \frac{\mu L_{\text{x}} \omega}{K_{\text{rc}}}. \quad (19)$$

The mean axial velocity of the liquid in the porous specimen  $w_{\text{m}}$  is correlated to filtration  $w$  by the ratio

$$\frac{\omega}{w_{\text{m}}} = \Pi. \quad (20)$$

If we substitute (19) into (18), take (20) into account, and transform with respect to  $w_{\text{m}}$ , we obtain

$$w_{\text{m}} = \frac{K_{\text{rc}} p_{\text{K}}}{\mu \Pi L_{\text{x}}}. \quad (21)$$

Under the conditions  $g \sin \varphi = 0$ , the metal-fiber specimen may be considered fully saturated from the beginning of absorption to the place of the impregnation front. Then with a view to Eqs. (4) and (17), Eq. (21) assumes the form

$$w_{\text{m}} = 2,19 \frac{\sigma \cos \Theta}{\mu} \frac{S_{\text{m}}}{L_{\text{x}}}, \quad (22)$$

where

$$S_{\text{m}} = \frac{d_{\text{B}}}{\Pi} \left[ \frac{1 - \Pi}{1 + (1 - \Pi)^2} - \frac{1 + \ln(1 - \Pi)}{2(1 - \Pi)} \right] \frac{(1 - \Pi)^3}{(1 - \Pi_{\text{lm}})^{1,5}} \exp \left[ -1,45 \frac{1 - \Pi}{(1 - \Pi_{\text{lm}})^{0,7}} \right].$$

It follows from (22) that the speed of capillary absorption (speed of the impregnation front) is proportional to the parameter of the liquid  $\sigma/\mu$ , the cosine of the boundary wetting angle, some function of the initial structural parameters, and inversely proportional to the length  $L_{\text{x}}$  from the beginning of impregnation.

The values of the boundary angles contained in (22) differ from the corresponding equilibrium values examined in [2] because the process of capillary absorption is nonsteady. For instance, in [7, 8] it was experimentally established that if in a state of equilibrium there is practically ideal wetting ( $\Theta \approx 0^\circ$ ), then in capillary absorption  $\Theta \approx 60-70^\circ$ .

The magnitude  $S_{\text{m}}$ , being the only structural characteristic of the kinetics of capillary impregnation of MFS when  $g \sin \varphi = 0$ , is a complex function of  $\Pi$ ,  $d_{\text{B}}$ ,  $l_{\text{B}}$ . An analysis shows that  $S_{\text{m}}$  as a function of  $\Pi$  has a maximum when the porosity is approximately equal to  $0.93 \Pi l_{\text{m}}$ .

The results of the experimental investigation of the kinetics of horizontal impregnation of MFS, partly presented in [3], confirmed the conclusions ensuing from an analysis of the dependence (22). The corresponding effect of the parameter of the liquid  $\sigma/\mu$  and of the complex  $S_{\text{m}}$  is confirmed. The inverse proportionality of  $w_{\text{m}}$  on  $L_{\text{x}}$ , i.e.,  $\partial L_{\text{x}}/\partial \tau \sim L_{\text{x}}^{-1}$ , manifests itself clearly. Then, as a result of integration we obtain  $L_{\text{x}} \sim \tau^{0,5}$ , which is fully confirmed by the primary results of the experiments.

As an example, Fig. 4 presents the experimental [3] and theoretical [by formula (22)] dependences of  $w_m$  on  $L_x$  for water and acetone for three oxidized specimens of MFS. On the basis of the above, it was assumed in the calculations that  $\cos \theta = \cos 60^\circ = 0.5$ . There is satisfactory agreement between the experimental data and the calculation.

#### NOTATION

$\Pi$ , porosity of the structure;  $\Pi_{lm}$ , limit porosity;  $d_B$ , fiber diameter;  $l_B$ , length;  $d_{\Pi t}$ , size of the contact spot;  $K_{KC}$ , coefficient of liquid permeability of the capillary structure;  $p_K$ , capillary pressure;  $\Delta p_l$ , viscous pressure losses in the liquid;  $w$ , filtration rate;  $w_m$ , mean axial speed of the liquid;  $L_x$ , distance from the beginning of absorption;  $\tau$ , time from the beginning of absorption;  $\varphi$ , angle between the longitudinal axis of the specimen and the horizontal plane;  $\theta$ , boundary wetting angle;  $\sigma$ , capillary constant of the liquid;  $\mu$ , dynamic coefficient of viscosity of the liquid;  $c$ ,  $c_1$ ,  $c_2$ ,  $c'$ , proportionality coefficients.

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#### HEAT EXCHANGE IN GAS FLOW THROUGH ROUGH PIPES WITH SURFACE SUCTION

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The article presents the results of determining the Nusselt numbers upon flow of nitrogen and helium through cermet pipes. The effect of roughness and of the suction of part of the flow on the Nusselt numbers is shown.

Turbulent flow through rough pipes with suction on the wall has not yet been sufficiently studied either theoretically or practically. There exist some works investigating heat exchange upon turbulent flow through a smooth pipe with surface suction [1, 2] but there are no data on heat exchange in rough pipes with suction.

The theoretical analysis of heat exchange upon turbulent gas flow in permeable channels is based on the idealized model of turbulence which does not quite correctly describe the real process, and it is difficult to use the results of the analysis in practice because they are very complicated. We therefore made an attempt

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